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Devoted to All Branches of Optics

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JANUARY 1917

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This Journal is the official organ of the Optical Society of America. It was founded to redeem the constitutional pledges of the society to promote the spirit of coöperation among workers in all branches of optics and coördinate the results of their labors.

To that end it publishes:

1) Original articles;
2) Short abstracts of articles of interest which have appeared in other journals;
3) Abstracts of patents;
4) The proceedings of the Optical Society of America and notes on the activities of local sections.

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Journal

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AND APPLIED OPTICS

Volume 1 JANUARY, 1917 Number 1

OPPORTUNITIES FOR RESEARCH

By F. K. RICHTMYER

The Optical Society of America is fortunate in having come into existence at a time when it is unnecessary to overcome the indifference—not to say prejudice—formerly experienced by some of the older scientific societies. The whole world today recognizes its debt to the scientist. The graduates of our universities need no longer look forward to teaching as the only career open to them, for biologists, chemists, physicists are demanded by the industries in increasing numbers. The growth of the industrial research laboratory is a matter of common knowledge.

Realizing the extent to which the advances of applied science depend directly on the advances of pure science, the Optical Society should early consider the means available for actively promoting research work in the several branches of Optics.

The Society itself cannot, as a society, engage directly in research. But there are several ways in which it can actively, through various standing committees, further the doing of research.

For example, there are, throughout the country, a large number of potential investigators who simply need encouragement, or the suggestion of some specific problem, to get a start on research. Many a promising

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young scientist, after graduation, has found a place as teacher in some smaller university, where, lacking the inspiration of co-workers, or perhaps suitable apparatus, he has gradually lost his enthusiasm and has become content to use his full time in the routine duties of teaching. There are over 500 colleges in the United States, but only a very small proportion of them can be classed as contributors to the work of the several scientific societies. One of the most potent means for utilizing, to the full, our research facilities is to take active and immediate steps to enlist the services of these men who, not being *required* to do research, are content to sit aside and watch others.

It is true that the university is the home of pure science. But the average university regards the doing of research not as a duty but as a privilege, freely or grudgingly extended, as the case may be. There are many who regard this attitude as inadequate—perhaps even unworthy of the university of today. The doing or research should be a part of the duties of every university professor, on a par in every way with teaching. The Optical Society can further the general recognition of this principle by creating on the part of those (at present) scientifically inactive teachers a demand for research facilities.

What is needed is suitable machinery for collecting a variety of specific (not general) research problems, and placing them before potential investigators, either through the columns of the JOURNAL or otherwise, with such individual encouragements as may be necessary to ensure a start on the work.

The collection of such problems should not be difficult in a society like the Optical Society, which is composed of both the pure science and the applied science groups of workers. We hear much today about the utilization of by-products. The by-products of any scientific investigation is a large number of problems more or less closely connected with the work in hand, but which must be absolutely passed by if the investigator is to proceed along the main lines of his investigation. These side issues, if they be followed out, frequently lead to results of quite as much importance as the original problem. The discovery of the phenomenon of photoelectricity by Hertz is a case in point. An important part of the report on any investigation should be a brief summary of such side problems as the investigator is willing to pass on to his fellow workers.

The increasing importance of the work of the scientist has imposed on him new obligations. What these obligations are, and how they can

best be met, are questions which are being considered by a number of scientific societies and organizations at work on the problem of utilizing and organizing our research facilities. Whatever be the lines along which such organization takes place, due recognition must be given to the fact that advance in science is almost exclusively due to the insatiable curiosity of the individual; on his desire to "learn the truth," regardless of what its ultimate value measured in material standards may be. Any organization of research must therefore leave the individual worker, once started, free to follow paths of his own choosing. Perhaps in this connection the phrase "co-operation in research" is preferable to "organization of research." The latter implies a central agency directing research toward a definite end. The former suggests the banding together of individuals whose interests are mutual. It is a case of the monarchy as contrasted with the democracy.

In other words, the Optical Society should not be content to hold meetings a few times a year and to publish a journal. Its opportunities extend far beyond these functions, which up to the present time have been the chief, if not the only, activities engaged in by many societies.

CORNELL UNIVERSITY
Ithaca, N. Y.
January, 1917

THE NATURE OF THE VISUAL RECEPTOR PROCESS[1]

By LEONARD THOMPSON TROLAND

I

The attempt to analyze the process of visual response into its constituent phases, and to determine the mechanism of these phases, makes progress by purely physical and inductive methods, so long as attention is confined to the relation between the external object and the image which is cast upon the retina. However, as soon as we proceed to inquire into the nature of processes of vision which lie posterior to the retinal image, we find ourselves forced to rely primarily upon the method of hypothesis and upon a study of the psycho-physical laws which correlate the dimensions of the stimulus (radiation) with those of our visual experience. The method of hypothesis becomes legitimate only when our speculations or inferences are given a form which is sufficiently definite to permit an accu-

[1] Presented before the Optical Society of America, at the New York meeting held December 28, 1916.

rate comparison of their consequences with the empirical relations which they purport to explain; and the use of psycho-physical functions in checking up such speculations is permissible only if due regard is paid to the extreme complexity of the mechanism upon which these functions must depend.

Over fifty more or less independent theories of the mechanism of vision are extant, although popular discussion of such theories generally limits itself to the speculations of Helmholtz and of Hering. The relative acceptability of the latter hypotheses, as compared with others, depends largely upon their extreme generality and their refusal to consider the details of the visual process. Speculations which have attempted to be more specific have been shown, in the majority of cases, to be inconsistent with special facts,[1] and consequently are no longer in the arena. Because of this rather than in spite of it, it seems to me advisable to bring forward further detailed hypotheses, which will no doubt perish as did their predecessors, but which may possibly leave behind them a progeny more vigorous than themselves. It is only by trial and error that we advance at all in efforts of this sort.

The Young-Helmholtz theory is primarily a theory of the receptor process, or of the mechanism by which radiation stimulates the ends of the optic nerve fibres. Hering's hypothesis, however, concerns the visual function at large, and employs the term *Netzhaut* to denote the entire visual nervous system. Nevertheless, it is consistent with both of these theories to suppose that the fundamental processes of the retinal rods and cones are photochemical, and it is highly probable that the final valid view will combine the fundamental features of both, as parts of a more detailed description of the distribution of visual functions among the various stages of the response. Consequently the considerations to be presented below need not be regarded as contradictory to classical opinions concerning vision, as in all probability they will finally appear to be mere extensions of the latter.

J. von Kries, in 1877,[2] attempted to give the Young-Helmholtz theory of the receptor process definite form by setting up hypothetical equations for the photochemical change occurring in the retina, and was able to show that the laws for retinal fatigue, deduced from these equations, coincided qualitatively, but not accurately, with the results of his experiments. For some reason, von Kries has made no further use of these equations in his later voluminous discussions of the visual process. Without being

[1] A notable exception to this rule is the so-called duplicity theory, which concerns the differential functioning of the rods and cones.

[2] von Kries, J., Ueber Ermüdung des Sehnervens. Archiv für Ophthalmologie (1877), 23, ii, 1–44.

aware of von Kries' attempts, the present writer essayed, in 1914,[3] to make certain seemingly necessary corrections of Hering's general theory of sensory response, and at the same time to give the doctrine mathematical form. The fundamental assumptions were similar to those of von Kries, although based upon a somewhat more detailed examination of possibilities. The consequences of the assumptions were also more completely developed.

The primitive theory of the process of stimulation represented by these two papers, postulates that the intensity of any "component" of visual sensation—*e. g.*, luminosity—is proportional to the crude rate of decomposition of a specific chemical substance located in the retinal rods or cones. This decomposition is supposed to follow a mass-action formula, in which the radiation enters as one of the mass constituents. As the reaction is regarded as irreversible, it is necessary to postulate, further, a repair process which provides the visual cells with new photo-sensitive substance by a simple principle of osmotic diffusion. The equations also include a term to represent the self-excitation of the cells, or the undoubted fact of their slight activity even in the absence of external stimuli.

The equation of total activity, on this basis, has the form

$$\frac{ds}{dt} = a - (c_d + c_k i)s, \tag{1}$$

where s is the concentration of the sensitive substance, S, in the cells, a the rate of restoration of this substance by diffusion, c_k the constant of self-excitation, c_d the constant of response, and i the intensity of the radiation. The "equation of compensation," which governs the repair process, is

$$a = r - c_r s, \tag{2}$$

where r is a constant depending upon the concentration of the sensitive substance, or its substrates, outside of the cell, and c_r is "the constant of repair."

It is shown, in the original paper, that these premises lead to conclusions which are in qualitative harmony with the psycho-physiological laws of visual and some other types of response. The theory avoids certain objections which are fatal to the Hering doctrine in its primitive form.

Several interesting consequences of the equations were not adequately discussed in the paper referred to. It was shown that the rate of decom-

[3] Troland, L. T., Adaptation and the Chemical Theory of Sensory Response. American Journal of Psychology (1914), 25, 500–527.

position of S when the receptor process is in equilibrium with a stimulus of intensity, i, should be

$$q = \frac{r(c_k + c_d i)}{(c_r + c_k + c_d i)} \tag{3}$$

which connects the quantity of excitation with the amount of stimulus, by an asymptotic law resembling in general form the recognized Weber-Fechner relationship. Analysis of this function shows that it accounts not only for the logarithmic factor in this latter relationship, but also explains the "upper" and "lower" "deviations" from the logarithmic law. By differentiating both sides of (3), we obtain

$$di = A(B^2 + 2Bc_d i + c_d^2 i^2)\, dq, \tag{4}$$

where $A = 1/c_r c_d r$, and $B = c_r + c_k$. If dq is regarded as a constant, as in the reasoning upon which Fechner's law is based, it will be perceived that for small values of i, the threshold, di, approaches a constant, AB^2dq. For middle values of i, the second term of the parenthesis will tend to be the governing factor, and this obviously tends to make di proportional to i, the classical Weber-Fechner relation, which is admitted to hold only for a mid range of intensities. For higher values of i, the last term will be the governing influence. Dividing (4) through by i, we obtain an expression for the fractional threshold, di/i, and by setting the differential of this expression equal to zero, we find that di/i is a minimum, when $i = B/c_d = (c_r + c_k)/c_d$.

Since the value of the stimulus intensity for which the fractional threshold is a minimum has been determined by experiment, this relation provides us with one datum which we might employ in attempting to calculate the actual chemical constants of the retina, assuming the correctness of our original premises. The recent measurements of Dr. Nutting on the "discrimination factor," indicate that di/i is a minimum for a retinal illumination of approximately 6.5 meter candles.

I have made attempts, by two other methods, to find some definite clues as to the magnitude of the retinal constants involved in the above theory, and at the same time to test the theory. Both of these methods relate to the law connecting the duration of an after-image with the length of the exposure used to produce the image.[4] When the field upon which the image is projected is of the same quality and intensity as that which

[4] See Troland, L. T., Apparent Brightness: Its Conditions and Properties. Transactions of the Illuminating Engineering Society (1917). Also The Laws of Visual Minuthesis, Journal of the Franklin Institute (1916), 181, 855–857.

was employed to generate the image, our assumptions demand the following relation between the "preëxposure," x, and the duration, y,

$$y = \frac{I}{D} \log e \left(\frac{1 - e^{-dk}}{D\Delta s} \right) - k, \tag{5}$$

where $D = c_r + c_k + c_d i$, Δs is the just noticeable difference in retinal sensitivity or adaptation, and k is an integration constant. By differentiating both sides of this equation it is possible to get an expression for D in terms of x and the slope of the curve for a given value of x. Applying this to an experimental curve, I found that D varies from 0.096 for short preexposures to 0.0114 for long ones. As D is, by definition, a constant this proves an error in the assumptions. However, there is some degree of correspondence of the theory with the facts, as for preëxposures ranging from 32 to 256 seconds, D varies only from 0.0287 to 0.0114. The value of i, in this case, was 1.8 meter candles.

If we employ a mean value of D = 0.020, and assume the constant of self-excitation, c_k, to be a small quantity, the above considerations permit us to calculate approximate values for c_r and c_d, which turn out to be equal to 0.016 and 0.0024, respectively.

My second attempt to determine the constants was perhaps even less justifiable in its method. It rested upon a solution of equation (5) for the special cases of $y = 0$ and $x = \infty$. I have been unable to reduce this equation to a form such that it shall be applicable to all cases. The two special cases in question, however, can be handled experimentally by the measurement of what I have called the "threshold preëxposure time," and the "equilibrium time" of a given visual stimulus.[5] If we neglect the constant of integration, k, we can obtain an expression involving these measures in which D is the only unknown. Graphical solution of this expression for $i = 0.62$ gives: $D = 0.041$ and $\Delta s = 0.051$. From this we obtain $c_r = 0.041$ and $c_d = 0.0063$, making use of the position of the Fechner's law minimum, as before. Placing k equal to zero assumes the existence of a level of sensitivity undoubtedly higher than that actually operative in the experiments. An expression involving the actual value of k (in accordance with our theory) can be written, but thus far I have been unable to find a solution for it.

I do not wish to have it supposed, for a moment, that I regard results such as those given above as possessing much ultimate significance, or as

[5] See Troland, L. T., The Laws of Visual Minuthesis, etc. Journal of the Franklin Institute (1916), 181, 579–581.

proving the assumptions upon which they partly rest. On the contrary, it is easy to show that although the laws developed from the theoretical assumptions have the same general form as those found by experiment, their quantitative fitting with the experimental curves is very poor. This is shown by the variation of the "constant," D, for different portions of the after-image life curve. The importance of these results lies primarily in the encouragement which they offer us to attempt further refinements, by the application of the same instruments of thought.

Even if we disregard the lack of quantitative correspondence between the results of the primitive chemical theory of sensory response and the facts, there are other considerations which show that this theory is based upon inadequate analysis. In the first place, it offers no explanation of the manner in which the excitation of the optic nerve fibres can be made to depend upon the rate of decomposition of the photochemical substance. Secondly, the supposition that the chemical change is irreversible is an improbable one, not only on general chemical grounds, but with reference to organic economy. The histological structure of the retina reveals no provision for the high metabolism and rapid nutritive diffusion process, which would be required to dispose of waste products and to supply new sensitive substance to the active receptor cells. In addition, the experimental fact of the restoration of the visual purple in the rods of an excised retina which has been bleached by exposure to light, gives an almost conclusive demonstration of the reversibility of the change, in one species of receptors. It has been suggested that, in the case of the visual purple, at least, the reaction is of the so-called pseudo-reversible type, [6] since restoration occurs only when portions of the pigmentary epithelium of the retina are left in contact with the rods. This epithelium may contain some substance which exerts a quasi-catalytic influence upon the restitutive change.

The simplest hypothesis to account for the stimulation of the nerve fibres is to suppose that this is due to the influence of the reaction end-products. If we regard the reaction as reversible and think of the end-products as being retained in the receptor cells, the degree of stimulation of the nerve may be supposed to depend upon the *concentration* of these end-products. For equilibrium conditions, this concentration would necessarily increase with increase in the light intensity.

The chemical effects which may be brought about by light are, of course, extremely varied in character. However, considering the electro-magnetic nature of light, it seems probable that the immediate effect is always some sort of ionization. When we consider the character of the nerve impulse,

[6] See Sheppard, S. E., Photo-Chemistry, 1914, 213–214.

and the electrical phenomena which are exhibited by the stimulated and unstimulated retina, it becomes reasonable to suppose that in the case of the visual receptors, ionization is the essential process. I attempted, in a crude way, to work out some of the consequences of this supposition, in a paper published in 1913.[7] The same point of view has been developed more in detail, but with regrettable neglect of elementary physiological facts, by P. Lasareff.[8]

The essential features of the *photo-ionic theory of the visual receptor process* are as follows: It is supposed that the retinal rods and cones contain certain electrolytes, or ionizable substances, which in the absence of light are ionized to an extent which is determined by the equilibrium constant of the reversible dissociation reaction. The introduction of light into the system alters the value of this constant by increasing the rate of the dissociating change without affecting that of reassociation. The equilibrium ionization thus increases with increasing light intensity.

If n and p be taken to represent the concentrations of the negative and positive ions, respectively, at an instant, and if c_n is the constant of the associative change, the net rate of change at any instant will be, on the simplest assumptions:

$$\frac{ds}{dt} = c_n np - c_d is - c_k s, \tag{6}$$

the remaining factors being defined as for the irreversible reaction. Assuming n = p, we have for equilibrium

$$n = \sqrt{\frac{c_d i + c_k}{c_n}}\, s. \tag{7}$$

If the total amount of sensitive substance, dissociated and undissociated, is constant, s_o, the equilibrium value of n will be represented by the expression:

$$n = G\left[\left(\frac{1}{4} + \frac{s_o}{G}\right)^{\frac{1}{2}} - \frac{1}{2}\right] \tag{8}$$

where $G = \frac{c_d i + c_k}{c_n}$, the equilibrium constant of the dissociation.

II

When one end of a wire is placed in contact with the cut end of a nerve and the other end is touched to the uninjured nerve sheath, a current flows from the sheath to the cut portion, indicating that the former is positive with respect to the latter. This is explained in the Nernst-Lillie theory of nerve

[7] Troland, L. T., A Definite Physico-Chemical Hypothesis to Explain Visual Response. American Journal of Physiology (1913), 32, 8–41.

[8] Lasareff, P., Theorie der Lichtreizung der Netzhaut beim Dunkelsehen. Archiv für die gesammte Physiologie (1913), 154, 459–469, and other articles.

action[9] by supposing that the nerve tissue contains an ionized substance the positive components of which diffuse through the sheath more readily than do the negative components.[10] In other words, the sheath has a *differential permeability* with respect to these ions. When the nerve is stimulated this so-called demarcation current suffers a decrease, or *negative variation*, which is accounted for by the Nernst-Lillie assumption that stimulation is always characterized by an increase in the general permeability of cell membranes, and thus a decrease in special differential permeabilities.

Membranes or interfaces through which positive and negative ions tend to diffuse at different rates will inevitably become the seats of potential differences or electrical double layers. This state of affairs is described as a *polarization* of the membrane. A general increase in the permeability of a membrane will, of course, tend to cause *depolarization*, but it is assumed in the Nernst-Lillie theory that depolarization will also cause an increase in permeability. It is this mutual interdependency of the two membrane factors which causes the state of stimulation to be propagated from one region of the membrane to outlying regions, as in the case of nerve conduction. It is clear that the membrane can easily be depolarized by sending an electric current through it in a direction opposite to its natural polarization, and it is this fact which is supposed to explain the universal stimulating capacity of the electric current. The same consideration accounts, in detail, for the definite laws governing such stimulation.

When the cut end of the optic nerve is connected by a wire to the retina, it is found that a current flows in a direction opposite to that which would have been expected.[11] This fact, that the retina is more negative than the nerve end, seems to indicate that the negative ions of the receptor cells diffuse more readily than do the positive ones, which is the reverse of the case for the nerve. When the retina is stimulated by light, the original potential difference is increased, instead of being decreased; in other words, the action of light is to augment the negativity of the receptor tissue. This is what would occur if light increased the ionization of the substance inside of the cells.[12]

[9] Lillie, R. S., The Relation of Stimulation and Conduction in Irritable Tissues to Changes in the Permeability of the Limiting Membranes. American Journal of Physiology (1911), 28, 197–223.

[10] The positive components are supposed to be hydrogen ions which, probably on account of their smallness, possess high mobility, and easily pass through molecular interspaces which exclude larger particles.

[11] See Fröhlich, F., Beitrage zur allgemeinem Physiologie der Sinnesorgane. Zeitschrift für Sinnesphysiologie (1913), 28, 28–165.

[12] It is possible that the increase in negativity due to the action of light on the receptor cells depends upon the ejection of electrons from atoms in the vicinity of the membrane, which is everywhere perpendicular to the electric and magnetic vectors of the light ray. The action would thus be strictly analogous to that occurring in the well-known photo-electric effect.

It is evident that if, in point of fact, the polarization of the receptor cells is opposite to that of the nerve fibres, there must be a constant tendency for these two tissue elements to depolarize each other, at their junction. In the case of the nerve, such depolarization will result in an impulse, which will pass along the fibre to the brain centers; in the case of the receptor cells, it will result in a momentary increase of permeability and the diffusion of a certain amount of substance through the cell membrane into the surrounding medium. After any single interaction of this sort, both the nerve fibre and the receptor cell will begin to recover their differential permeability as soon as the depolarization has reached its maximum. The period of recovery corresponds with the well-known "refractory phase," during which all stimuli are ineffective, which is characteristic of all irritable tissues. Since the length of the refractory phase for nervous tissue is very small—often less than 0.001 second—it is not unreasonable to suppose that the nerve fibre completes its recovery before the receptor cell has been able to return to its initial state. The reëstablished differential permeability and polarization of the nerve fibre consequently is maintained until the differential permeability of the receptor cell reaches, in the course of its recovery, a point at which the resulting receptor polarization is again able to neutralize that of the nerve fibre, and thus to cause a second depolarization of the latter. This intermittent action will obviously continue *ad infinitum*, if the electrolytes of the tissues are kept at the requisite concentrations.

The hypothetical mechanism of the interaction of receptor cell and nerve fibre sketched out above is capable of providing solutions for two very perplexing enigmas regarding visual response. There are two fundamental laws of nerve physiology which appear, superficially, to be in direct contradiction to obvious facts of vision. In the first place, DuBois Reymond's law of stimulation states that a nerve fibre is excited only by *changes* in the strength of the electric current which acts upon it, a constant stimulus being without effect. This seems to be contrary to the fact that visual sensations of long duration can be obtained by the use of constant stimuli, the phenomena of fatigue notwithstanding. Secondly, there is now a large body of evidence to support the general application of the "all or none" principle to the excitation of nerves.[13] This principle asserts that if a nerve is stimulated at all, it is *ipso facto* stimulated to its maximal capacity; a threshold stimulus produces as great a nervous response as any possible stimulus; a law seemingly out of harmony with our ability to perceive delicate gradations in brightness. These two laws would appear

[13] See Verworn, M., Irritability, 1913, 139–146. The "all or none" law obviously necessitates a "quantum theory"—of nerve action.

to require that our visual experience should consist of a discontinuous series of flashes, always in the form of black and white silhouettes.

It is evident, however, that a receptor process consisting of an oscillatory discharge, the frequency of which increases with the intensity of the stimulus, will clear up both of these dilemmas at one sweep. Such a process translates a constant stimulus into intermittent action upon the nerve, but makes the number of impulses sent along the nerve in unit time depend upon the magnitude of the stimulus. There is thus a continued transmission of energy to the brain centers, and the amount transmitted is controlled by the force acting upon the receptor. As a general answer to the questions raised by the establishment of the "all or none" law for nerve action, this conception of the mode of conduction of intensity variations is not new.[14]

The manner in which the number of unit pulses sent along the nerve fibre per second can depend upon the degree of ionization of the specific substances of the receptor cells is perhaps too obvious to need discussion. The known facts with regard to "all or none" action and refractory period seem to indicate that the rate of break-down and recovery of a membrane is nearly independent of the electrical forces which are acting upon it. However, the degree of recovery which is necessary to permit the development of a critical polarization must depend upon the concentration of ions in the cell which the membrane surrounds. Hence this threshold polarization will be reached more quickly when the ionization, n (Equation 8), has a high value than when it has a low value. Since the ionization constant increases with increase in the light intensity, the number of impulses started along the nerve per second must bear a similar relation to the stimulus magnitude. The recovery of the membrane probably follows an asymptotic law (with respect to the time), so that the frequency would not be *proportional* to the ionization, except approximately so for relatively high intensities of stimulation.

Although the hypothesis above outlined has been developed primarily to meet the exigencies of a theoretical situation, it is possible to appeal to more direct evidence for its further validation. The recent studies reported by F. Fröhlich[15] on the action currents of the cephalopod eye indicate that the retinal process in this organism has an oscillatory character. The oscillations observed by Fröhlich vary in a regular manner with the intensity of the stimulus and with the wave-length, in such a way as to exhibit a higher frequency for the higher response values. The frequencies actually measured varied from 20 to 90 per second. The existence of

[14] See Forbes, A., and Gregg, A., Electrical Studies in Mammalian Reflexes, II. American Journal of Physiology (1915), 39, 225–232.

[15] *Loc. cit.*

analogous oscillations in human vision is indicated by the phenomenon of "Charpentier's bands," a series of dark lines which appear on a white area, parallel to an edge which is moving into a dark field. According to their discoverer the oscillations required to account for these bands would have a frequency of about 36 a second. Exactly how this frequency is related with the intensity of the stimulus has not been determined, but the general properties of the bands are in harmony with their explanation in terms of an intermittent receptor process.

The results of the histological examination of the retinal receptor cells are consistent with the theory of their functioning which I have suggested. The rods and cones, proper, are made up of a substance quite different from that of the conducting portion of the tissue. They are separated from what is probably the beginning of this conducting portion by a thin plate known as the "intermediate disk." The fact that they are possessed of a high index of refraction indicates that the molecules of the substances of which they are composed respond strongly to light, and their double refractivity points to a regular arrangement of the axes of these molecules, possibly with reference to the efficiency of their response to the electro-magnetic waves, the force vectors of which must necessarily be at right angles to the length of the rod or cone. The transverse septa which occur in both the rods and cones at intervals of about 0.5 μ (0.0005 mm) may be primarily skeletal structures, but they may also serve to prevent or to control diffusion within the cells.

I have thus far made no mention of the problem of color vision, or of the probable mechanism by which the retina responds selectively to radiation of different wave-lengths, and of the means which enables the nerve to transmit the specific results of this response to the brain. This problem does not arise in the case of the rods, since they are color blind. In one sense, however, they do respond selectively to different wave-lengths, since the intensity of their action in the various parts of a uniform energy spectrum varies in accordance with their "visibility curve," which undoubtedly depends upon the law of molecular resonance of the special photochemical substance which they contain. As was recognized by Thomas Young in 1802, to explain the facts of color vision, we have only to assume the existence in the retinal receptors for color (the cones) of three substances of this sort, having the maxima of their resonance curves in three representative regions of the spectrum, and to suppose that the relative intensities of these three excitations, for any given wave-length, can be transmitted along the nerve to the brain.

If three such substances exist, it would be natural to suppose that the visibility curve of the cones represents the sum of the three activities. However, as I have recently pointed out,[16] the extreme symmetry of the cone visibility curve, especially when corrected for the selective absorption of the ocular media, seems strongly to suggest that a single resonance process underlies the impression of brightness, for cone vision as well as for rod vision. There are other important considerations which indicate that the cone luminosity process is independent of the specific color receptive process.

The most obvious hypothesis to account for the transmission of color quality (or its determinant) along a nerve fibre would be to attribute this transmission to control of the *wave form* of the nerve impulse. Since the facts supporting the "all or none" law make it probable that the individual impulse is invariable in form as well as intensity, it would be necessary to suppose that the variable "wave-form" in question consists in a special grouping of impulses in time and space, after an analogy with a telegraph alphabet made up only of "dots." Intensity transmission would then depend upon the total number of "unitary groups," passing through a cross-section of the nerve in a given time, while quality transmission would be determined by the arrangement of the individual impulses in a single group.

How such a grouping of the impulses can be accomplished is not entirely clear. If we could legitimately postulate the existence among the cones of four types of receptors, one for luminosity, and one for each of the fundamental colors of the Young-Helmholtz theory, it would be fairly easy to conceive a mechanism to accomplish the desired result. However, in the face of accepted empirical facts (which are none too analytically interpreted in the majority of discussions of the subject), we seem forced to believe that the cones are all alike in their function, except for a possible general regional differentiation which is of no immediate significance for our problem. It is still possible that the individual cones are stratified with respect to their selective response to light, so that each of them is really three or four receptors in one. This possibility was distinctly suggested by certain experiments of König and Zumpft,[17] but their work seems to have been repudiated. However, the conception is in harmony with the fact that the increase in the length of the cones, which is found in passing from the periphery to the center of the retina, is paralleled by an increase in their capacity to register color differences. Whether or not the conical form of

[16] Trans. I. E. S., *loc. cit.*

[17] König, A., and Zumpft, Ueber die lichtempfindliche Schicht in der Netzhaut des menschlichen Auges. Königs gesammte Abhandlungen, 333–338.

the cones is related with any differentiation of substance from base to tip, is a question which at present has no satisfactory answer, but the rod-like form of the foveal "cones" suggests that the function of the cone figure is structural rigidity, rather than differentiation of response.

Among the varied multitude of speculations which have been advanced to explain visual response, there is at least one possible hypothesis which is missing. This is the idea that radiation may act directly and specifically upon the permeability of the *membranes* of the receptor cells. By such action it would be possible to obtain a specific control of the rhythm of discharge of these cells with reference to the wave-length of the stimulus. As yet, however, I have made no detailed study of the potentialities of this hypothesis.

HARVARD UNIVERSITY
December 22, 1916

THE PETROGRAPHIC MICROSCOPE

A USEFUL TOOL IN APPLIED OPTICS[1]

By FRED E. WRIGHT

In research work progress depends in many instances on the kind and quality of apparatus available. Great discoveries have, of course, been made by such men of genius as Newton, Galileo and Faraday, with exceedingly crude apparatus; but, as a general rule, advancement has resulted chiefly from the introduction of new methods and new apparatus adapted to the solution of definite problems. The investigator uses the apparatus at hand so far as it serves his purpose, but he may soon reach a point where the requirements of his problem are such that the old apparatus is no longer adequate and must be replaced by more suitable and powerful weapons of attack. A long series of careful measurements with a certain type of apparatus may be discarded as obsolete the moment new instruments and methods are to be had which afford greater precision and deeper insight into the problem. This has happened repeatedly in optics and is in fact the common experience of every research student in exact science. The pity of unsatisfactory apparatus is that work done with it may not be reproducible and hence is wasted and has to be undertaken anew with proper apparatus.

In view of the importance to optics of new or improved instruments and of methods of attack, it is desirable that the Journal of the Optical

[1] Presented before the Optical Society of America at the New York meeting, held December 28, 1916.

Society of America include papers covering this subject. A new method or a new tool devised for a special purpose in one branch of optics may serve well for other purposes in another branch. The description of such an instrument or method in a journal whose readers have scientific interests in common, renders it at once generally available, in principle at least, and may save the reader much time and energy, as he might otherwise be required to invent for his own needs an instrument essentially the same as the one described. It is with this purpose in view, to direct the attention of workers in applied optics to the petrographic microscope as a useful tool in many problems that the present paper has been prepared.

The petrographic microscope was introduced half a century ago by geologists because chemical and mineralogical methods failed to furnish adequate information regarding the rocks. To know a rock thoroughly it was essential to know not only its chemical composition but also its mineral composition and its texture as expressed in the spacial grouping of the mineral components. This knowledge was obtainable only through the aid of the polarizing microscope, or petrographic microscope as it is now generally called.

In recent years the effort has been made, especially at this laboratory, to study from the standpoint of experimental geology the conditions of formation of minerals, rocks, and ores. This implies the artificial reproduction of these minerals and rocks and their identification under the petrographic microscope. In such work the time element is a most important factor. Rocks in nature form slowly and the crystals which develop have ample opportunity to grow to a fairly large size. In the laboratory, on the other hand, time is limited, the furnace charges are small, and the resulting crystallized aggregates are very fine grained. This has of necessity led to the development of petrographic microscope methods suitable for the accurate measurement of the optical constants of crystal grains measuring only 0.01 mm in diameter. With these methods it is now possible to measure readily and with sufficient accuracy the optical constants of a substance in particles so fine that in water they settle at the rate of only 30 cm an hour. As the diameter of the particles in most materials and powders exceeds 0.01 mm (1/2500 inch), it is evident that this downward extension of the limits, to which petrographic microscope methods can be applied, has greatly increased the range of application and usefulness of the petrographic microscope.

In this connection it may be well to emphasize the fundamental difference between the petrographic microscope and the ordinary chemical or biological microscope which is essentially a magnifier and serves to render visible minute objects and details that would otherwise escape attention. Such objects are recognized by their shape, their size, and their color. In the case of bacteria the differences in their behavior toward certain stains aid in the diagnosis, while in other lines of investigation micro-chemical reactions serve the same purpose. The petrographic microscope, on the other hand, is not only a magnifier but also, and essentially, an optical instrument of precision for the measurement of the optical constants of substances either in thin plates or in powder form.

To illustrate this difference between the two-types of microscopes by an example: Quartz (SiO_2) is a colorless, transparent mineral crystallizing in hexagonal prisms and pyramids of such characteristic shape that this feature alone, when present, is sufficient to establish its identity. If quartz appeared invariably in such well developed crystals, the crystal habit would suffice for its determination and the ordinary microscope would answer the purpose well; and if all substances had different and equally characteristic crystal shapes, the crystal form alone would constitute an adequate basis for distinguishing substances. But this is of course not the case, for we can destroy external crystal form by crushing the crystal into bits. Each fragment of a quartz crystal is still quartz in spite of its irregular form. Under these conditions, other properties inherent in quartz itself, and not dependent on its outer form, must serve for the determination. These are preëminently the optical properties, which serve to distinguish with certainty between different substances, especially crystallized substances.

In a crystal the component atoms are arranged in orderly fashion as the result, during crystallization, of the action of the interatomic forces. These forces are different in different directions and are vectorial and polar in character; they find expression not only in crystal form, but also in the vectorial behavior of a crystal toward any external system of forces, such as light waves acting upon it. The physical properties of crystals vary in such definite manner with the direction that these properties and their variations serve as diagnostic features in the determination of crystallized substances. Thus on quartz fragments we can measure with the petrographic microscope the refractive indices for different wave lengths of light, the differences in refractive indices for any given wave length (bire-

fringence), and the character and shape of the optic ellipsoid to which the several optical properties are referable. These data of measurement serve to render the determination of the fragments as quartz practically certain. It may be noted that there are only four optical properties of crystals which are ordinarily measured in determinative work and are expressed as numerical optical constants of a given substance. These are: the principal refractive indices, the principal birefringences, extinction angles, and the optic axial angle. Other optical properties of diagnostic value, as color, absorption, pleochroism, elongation, optical character, dispersion of the optic axes, etc., are stated in more or less qualitative form.

Of these properties the refractive indices are the most important and fundamental and also the easiest to measure with the petrographic microscope. The refractive index is the ratio of the velocity of light in air to that in the given substance. The fact that different substances have different refractive indices enables us to use the refractive index as a basis for classification. For a given wave length of light, a substance which crystallizes in the isometric system has only one refractive index; this index is the same for all directions in the substance and the optical surface of reference is a sphere. In other substances, as those of the hexagonal and tetragonal systems, the optical surface of reference is an ellipsoid of rotation; such crystals have two principal refractive indices. In still other crystals (orthorhombic, monoclinic, triclinic) there are three principal refractive indices and the surface of reference is a triaxial ellipsoid.

In the practical measurement of the refractive indices of the grains in a powder, the particles are immersed in liquids of known refringence, and the refractive index of any given grain is compared with that of the enveloping liquid. These grains are more or less lenticular or prismatic in shape and when immersed in a drop of liquid on an object glass and placed on the microscope stage, give rise to the phenomena of prismatic refraction. Under conditions of oblique or central illumination these phenomena are easily recognizable; and relative refringence determinations between liquid and mineral grains can be made with an accuracy of ± 0.001 under favorable conditions. This method requires a set of standard refractive liquids. A convenient set is one of mixtures so prepared that the refractive index of each successive liquid is 0.005 higher than that of the liquid preceding it.

The birefringence, another useful constant, is the difference between the principal refractive indices; it signifies a difference in the rate of transmission for plane-polarized light waves vibrating along different directions

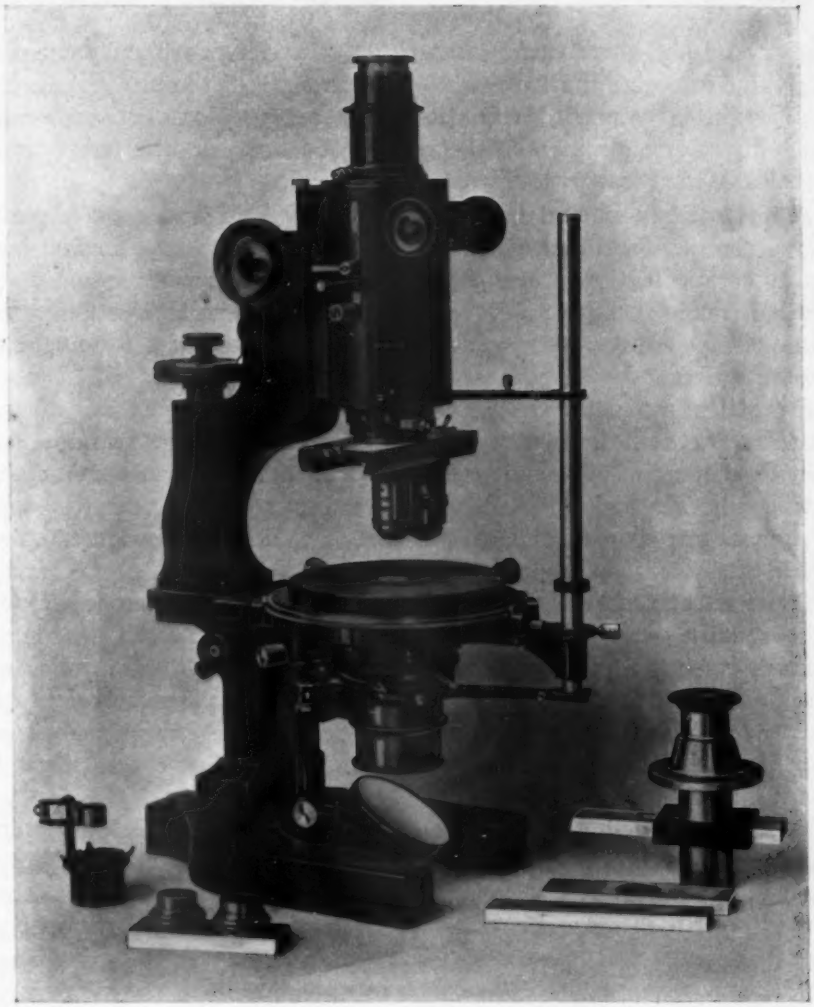


Fig. 1

Research model Petrographic Microscope constructed by Bausch & Lomb Optical Company, Rochester, N. Y. Distinctive features of this microscope are: large Ahrens polarizing prism; sensitive tint plate mounted in a sliding carriage revolvable about the axis; aplanatic condenser Na = 1.4; substage apertometer; rigid bar connection for simultaneous rotation of nicols; sliding adjustable mount for two objectives, so adjusted that if one objective is in focus and centered, the second on insertion is also centered and focused; combination wedge; negative and positive lenses below and above the polarizer for the correction of astigmatism; swingout lens above Bertrand lens to insure imaging of object plane in plane of iris diaphragm in draw-tube; universal eyepiece with tenth millimeter co-ordinate scale, graduated quartz compensator, and biquartz wedge plate; small prism mounted in slider below eyepiece for observation of interference figures by the Lasaulx method; sliding diaphragm with eyepiece for observation of interference figures of small mineral grains. (19)

in the crystal and this in turn gives rise to interference phenomena which are of value in determinative work. The strength of the birefringence of a crystal grain is estimated from its interference colors between crossed nicols. In colorless minerals like quartz the scale of interference colors is practically identical with Newton's color scale.

A birefracting mineral becomes alternately light and dark four times during a complete rotation of the microscope stage between crossed nicols. In many crystal sections these positions of darkness or extinction, as they are called, make definite angles with a crystallographic direction such as the trace of a cleavage plane. These are then called extinction angles and are a useful diagnostic feature.

A further important constant is the optic axial angle. Its measurement includes the determination of the angle between the two optic axes, or binormals, and also the determination of the character of the acute bisectrix, whether α or γ, from which follows by definition the optical character, negative or positive, of the mineral. For most purposes a rough estimate of the optic axial angle is sufficient.

These are the more important of the several optical phenomena which can be readily determined with the petrographic microscope on grains measuring at least 0.01 mm in diameter. They may be used in the identification of any given substance, especially crystal grains or mixtures of different crystal fragments. In many cases the determination of only two or three of these constants is sufficient to establish the identity of a substance.

Experience has shown that it requires practice to apply petrographic microscope methods most expeditiously. In this respect the petrographic microscope is analogous to the lathe in the machine shop. The lathe in the hands of an instrument maker is a most successful instrument, and remarkable results can be accomplished with it; and yet the principles on which it is based are not difficult to grasp. Similarly the petrographic microscope is equally useful in its line, and its methods are simple and easy to apply for one who has had the necessary practice. One of the most difficult features of such work is the necessity of thinking in space. The optical properties of crystals vary with the direction, and the observer must picture to himself the spacial relations involved if he would understand and take advantage of the phenomena he observes. This means, of course, a certain amount of training on the part of the chemist who would utilize this tool, but the saving of time and expense thereby gained more than com-

pensates for the labor involved. Determinations with the petrographic microscope are commonly made in a few moments; chemical determinations, on the other hand, of similar nature may require several hours or even days, and may not then furnish results which can be interpreted. It is fortunate that in most problems the determination of the refractive indices alone suffices for the diagnosis, and that the method used for such determination is extremely simple and can be accomplished with the simplest type of petrographic microscope.

For the accurate and rapid determination, however, of all the optical properties of fine crystal grains or plates, it is advantageous to use a microscope specially constructed for the purpose. Such an instrument should be so designed that it meets the requirements of an instrument of precision, that proper adjustment facilities are incorporated, and that attachments are included which enable the observer to make a series of measurements of known accuracy with the least expenditure of time and energy. The microscope illustrated in Fig. 1 is built with these principles in mind, and has been found satisfactory in practical routine work.[2] With this microscope it is possible to measure the optical properties of minute mineral grains and of crystal plates in the thin section with a degree of accuracy heretofore attainable only on much coarser grains and plates and with a far greater expenditure of time.

In the field of applied optics, the petrographic microscope has proved useful in the examination and testing of: a) raw materials for the manufacture of glass; b) abrasives and the action of different kinds of abrasives and polishing materials; c) cements; d) paints and enamels; e) purity and homogeneity of crystallized chemicals employed in the laboratory and in manufacturing processes; f) "stones" and crystallization phenomena in glass; g) pots and other containers for molten glass to be used at high temperatures; crystallization phenomena in such pots and causes of failure of same in actual use. In short the petrographic microscope may be used to advantage in the determination of non-metallic materials, whether crystallized or amorphous, in individual units or in aggregates, coarse or fine grained, and of organic or inorganic nature. Refractive indices can be ascertained in grains a few microns thick, and other optical properties on grains 0.01 mm or greater in diameter.

[2] Described in part in American Journal of Science (4) 29, 407–414, 1910; also in "The Methods of Petrographic Microscope Research," Carnegie Institution of Washington, Publication 158, 1911. New research model constructed by Bausch & Lomb Optical Co. of Rochester, N. Y. Later improvements are described in J. Wash. Acad. Sci. 6, 465–471, 1916.

GEOPHYSICAL LABORATORY
Washington, D. C.
January, 1917

THE OPTICAL PROPERTIES OF LIGHT FILTERS[1]

By C. E. K. Mees

Light filters are used in photography to modify the color rendering obtained in the translation of color into monochrome and especially in photo-engraving in order to obtain the three selection negatives from which the printing plates are made by means of which color prints are obtained, these three selection negatives being commonly made through orange-red, green and blue-violet color filters.

The optical effects produced by the use of such color filters may be divided into two classes. The first class includes those resulting from the use of a theoretically perfect filter—that is, a filter of which the faces are strictly plane parallel and which is free from strain; if a set of filters be considered all three filters are assumed not only to have plane parallel faces but to be of the same thickness. The second class of effects includes those which result from imperfections occurring in the filters—that is, from departures from the conditions of plane parallelism in the faces and of equality in thickness between the three filters of a set.

Part I

The first effects, namely those inherent in the use of color filters when these color filters may be considered to be composed of perfectly plane parallel pieces of optical glass, have been discussed by Dr. R. S. Clay in the Process Year Book.[2]

1) Let us first consider the effect of a plane parallel filter on the aberrations of a lens system.

Suppose that we place in front of a lens a plane parallel piece of glass, then it will produce upon the oblique rays a displacement compared with the axial beam which will result in two effects: a) it will cause a species of spherical aberration all over the field, causing the marginal rays of each image forming pencil to focus at a greater distance from the lens that its central ray; and b) on account of the tendency of a small oblique pencil to foreshorten abnormally in the object plane, the image plane will be curved backward. These displacements are greatest when the obliquity of the rays is greatest; they also depend upon the thickness of the filters. They will therefore be at a maximum when using a thick filter behind a

[1] Communication No. 41 from the Research Laboratory of the Eastman Kodak Company.

[2] 1903-04, page 60; 1904-05, page 92.

short focus lens working at a wide angle and a high speed, and they will be minimized by the use of thin filters behind long focus lenses and by placing the filter in front of the lens when the image is smaller than the object.

Given a Process lens, working at *f*:63 of 16″ focal length, taking a picture 10″ × 12″ and forming an image one-half the size of the object. To find the spherical aberration and curvature of field produced by interposing in front of the objective a filter 15 mm thick and of an index of refraction 1.55.

The distance of object from the center of the lens will be 48″ = 1200 mm; from the filter about 1175 mm, depending upon the length of the lens. The clear aperture of the lens will be 64 mm.

Taylor finds, System of Applied Optics, page 80, the longitudinal aberration in the object plane of any pencil radiating from a point in the axis at a distance u[1] from the filter:

$$\Delta u' = -\frac{n^2 - 1}{2n^3}\left(\frac{h}{u}\right)^2 d$$

n = index of refraction of filter, h = distance from axis at which the pencil strikes the filter; for an extreme ray, in our case, $h = 32$, also $u = 1175$ mm. We find for $\Delta u'$

$$\Delta u' = -0.0021 \text{ mm.}$$

In the image plane the longitudinal aberration is $= -\frac{\Delta u'}{4} = +0.00052$ mm.

To get the extreme diameter of the circle of confusion, the longitudinal aberration must be multiplied by $\frac{64}{600}$;

$$\therefore \text{diameter} = 0.000056 \text{ mm.}$$

For an aperture ratio of *f*:20 it will be reduced to $\frac{1}{10}$ this value.

[1] The designations in this article have, with the permission of the author, been changed in accordance with the suggestions on designations made in another article in this issue. $\Delta u'$ stands for $u_u' - u_s'$, the difference between the paraxial u′ and the u′ belonging to the zone at the distance h from the axis.

The longitudinal aberration of an oblique pencil making an angle a with the axis is, from Taylor, page 180,

$$\Delta u' = -\frac{n^2 - 1}{2n^3} d3 \tan^2 a = 9 \tan a$$

$$\text{maximum } \tan a = \frac{6.5}{24} = 0.27 \quad \tan^2 a = 0.073;$$

$$\therefore \Delta u' = -0.66 \text{ mm.}$$

For this oblique pencil, the focus is set back $\frac{1}{4}\ \Delta u'$ or $+$ 0.16 mm. The diameter of the corresponding circle of confusion

$$d = 0.016 \text{ mm,}$$

which is far below or within the limit of tolerance.

2) Secondly, let us consider the effect of a set of perfect filters on the size of image produced.

If for the three colors we use filters of the same thickness, then we shall obtain a difference in the size of the images produced owing to differences in the refractive index of the glass for different colors of light; that is to say, the effective thickness will be altered by the dispersion of the glass. The amount of this, dd, can be calculated for a given glass, knowing its refractive index and dispersion. The formula is

$$\mathrm{d}d' = \frac{\mathrm{d}n}{n^2} d$$

where n is the index, dn the dispersion, and d the filter thickness. For a typical glass used for filters, and taking a filter thickness of 15 mm, the difference in refractive index between the red and the green was 0.0068 and between the green and the blue 0.0159, which gave an effective difference of path between red and green of 0.043 mm and between red and blue of 0.142 mm. Such a filter will not be used with a smaller optical path than 50 cm, and in general, if used in front of the lens, the distance from the object plane to the nodal plane of the lens will be a meter or more, and the effect will therefore produce a difference in size between the red and blue images of the order of one part in 5000, an amount which is much smaller

than the resolving power of the half-tone screen used in photo-engraving and which in practice is indistinguishable. This effect, therefore, due to the dispersion of a filter for different colors, may be considered negligible and it is unnecessary to compensate for it. It could be compensated either by using the three filters of slightly different thickness to produce the necessary compensation, or by refocusing to produce the correction required, but photo-engraving cameras are built scarcely rigidly enough to obtain a correction by refocusing, and it would involve a considerable additional cost to have the glasses ground to correct the thickness of the three filters separately while in practice the errors introduced by the fact that the glass surfaces are not perfectly plane parallel necessarily obscure these effects dealt with by Dr. Clay and which are inherent in the use of plane parallel filters.

Part II

The effects upon the definition and size of image introduced by the use of filters which are imperfect either in plane parallelism of surface or in equality of thickness in a set of filters are naturally very much more complicated than those which result from the use of perfect filters. Since the errors of surface may be of any type—wavy, spherical, wedged, or cylindrical—the investigation of the effects produced can only be performed experimentally. It might be thought that the errors introduced by such imperfect surfaces can best be observed by an interference method. A trial in this way led to a curious discovery.

While the interference figure is a very rigid test of a nearly perfect surface, it is not at all a good guide as to the effect which will be produced upon a lens image by the comparatively imperfect surfaces occurring in actual filters. When a proper experimental method had been worked out, it was found that some filters which gave quite perfect images with lenses appeared poor in an interference test, while other glasses which showed much better on the interference test were useless for the purpose of light filters upon long focus lenses. This being so, it was found necessary to test the filter by the observation of its effect upon a lens image. Since the aberrations introduced by a filter increase very rapidly with the increasing focal length of the lens on which it is used, a telescope objective of about 60 mm diameter and about 1200 mm equivalent focal length was selected and placed 7 m from a test object which was illuminated by a lamp, the image of the test object produced being examined by means of a

10 power eyepiece. The test object used consisted simply of a number of opaque lines crossing each other so as to leave transparent squares, the

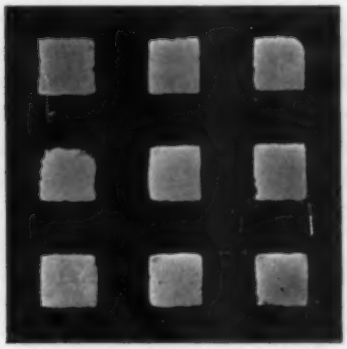


Fig. 1

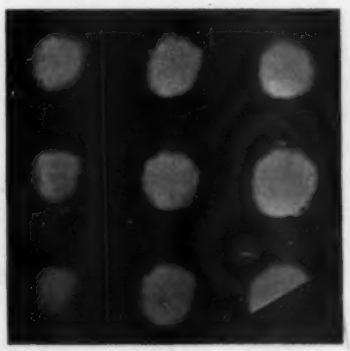


Fig. 2

object being shown magnified in Fig. 1. In a filter made in second grade glass which showed strain and cylindrical curvature, so that it introduces astigmatism into the image, the test object appears as is shown in Fig. 2. A good filter in this grade of glass will give an image as shown in Fig. 3,

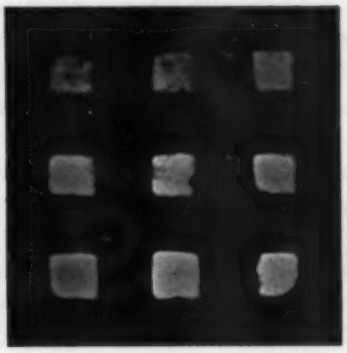


Fig. 3

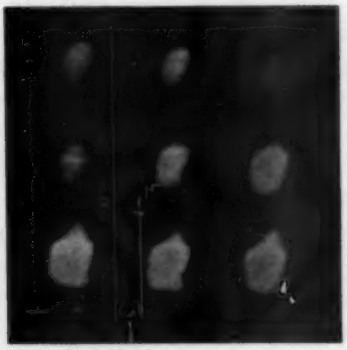


Fig. 4

but this image can easily be deformed by pressure, as, for instance, by fastening a circular filter too tightly in its cell, with the production of the effect shown in Fig. 4. In some filters examined the cylindrical curvature was so great that lines running at right angles to each other differed in

their focal points by as much as 10 cm, one set of lines being in focus at one point and the other at the other, as is shown in Fig 5, which represents the two focal points of such an astigmatic filter.

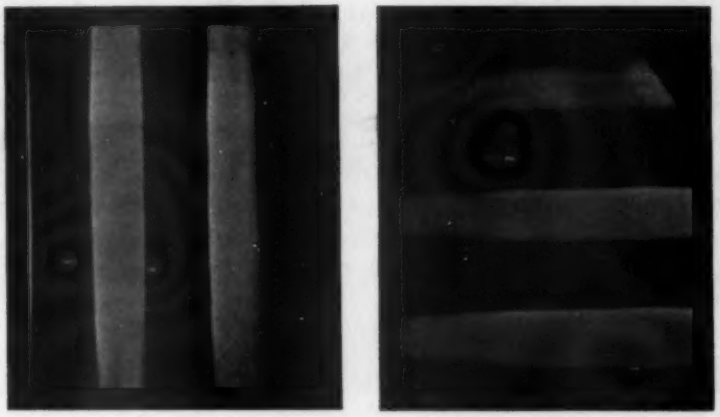


Fig. 5

Filters of the first grade should of course show no effect whatever upon this comparatively coarse test object, and filters cemented in optical flats cannot therefore be tested upon it in order to see whether they are really satisfactory, since filters cemented in optical flats are intended for use on very long focus lenses such as those used in photo-engraving, while the filters in the second grade of glass are not intended for use on lenses of greater focal length than 30 cm.

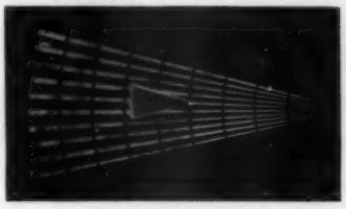


Fig. 6

In order to test the high grade filters cemented in flats the special resolution test object shown in Fig. 6 was used, Fig. 6 being a much magni-

fied image of the test object. It will be observed that this consists of two converging fans, the one 1/10th of the diameter of the other, and the test consists of observing the resolution of the lines in the inside fan. With the full 60 mm opening of the telescope objective and no filter in position the small fan is resolved through about half its length, and a lowering of the aperture of the lens from 60 mm to 50 mm is very visible indeed in its effect upon the resolution; the lens being thus seen to be almost entirely free from central aberrations.

This instrument is not only used for observing the definition given by filters, but also for measuring their effect upon the focal length of the long focus lens employed, thus detecting at once any spherical curvature in the surfaces. This subject of focal length or power of the filter will be referred to again shortly.

Part III

The differences in the size of the images produced by a set of three-color filters which differ slightly in thickness and also in focal length due to cylindrical or spherical curvature of the faces might be thought to be subject to accurate calculation, but the effects are so complicated that it was thought desirable to design an instrument for directly measuring these differences.

An instrument was therefore constructed in which a lens of about 25 cm focal length formed two images about 20 cm apart of two cross hairs, the exact distance between the images being measured by means of microscopes mounted on carriages actuated by micrometer screws. The various filters could therefore be placed over the lens, and the effect of the filter upon the size of image could be directly determined and compared with the amount calculated from the focal length and the thickness. Using this instrument the following results were obtained:

(a) Effect of difference of thickness in truly plane parallel filters: Taking refractive index of the glass as 1.54, we find

	that	$dp = 0.35\ dd$
where		dp is difference in optical path
	and	dd is difference in thickness of the filter

$$\therefore \quad dy' = \frac{0.35\ y'\ dd}{u}$$

where dy' = difference in size of image

y' = size of image

u = distance of object plane

In our instrument $y' = 189$ mm

$u = 708$ mm

$$dy' = \frac{0.35\ dd \times 189}{708}$$

$$= 0.093\ dd$$

In an actual measured case

dd was 0.76 mm

and dy' was found to be 0.061 mm

so that $dy' = 0.080\ dd$,

thus agreeing well with the calculated result found above.

(b) Effect on the size of image of spherical surfaces on the filters, producing a lens of power F, where $F = \frac{1}{f}$ and

f is the true focal length of the filter alone.

This is measured on the definition testing instrument as a difference in the focal length of the testing lens produced by the interposition of the filter.

Measuring on a lens of focal length f_1 and with a filter giving a difference of focal length df

$$F = -\frac{df}{f_1^2}$$

Now, the alteration of image size using such a spherical filter behind a lens is $\frac{i}{y'} F$ where i is the distance between the filter and the nodal point of the lens.

With our definition testing instrument $f' = 1500$ mm

$$\text{and } F = \frac{df}{(1500)^2}$$

In the magnification instrument $i = 30$ mm

$$\text{and } \therefore \frac{dy'}{y'} = F.\ i = \frac{30\ df}{(1500)^2}$$

For an actual pair of filters $df = 37$ mm

$$y' = 189 \text{ mm}$$

$$\therefore dy' = \frac{189 \times 37 \times 30}{(1500)^2}$$

$$= 0.094 \text{ mm}$$

dy' found by measurement was 0.112 mm.

It may be remarked that two filters may be of the same power, but if in one filter the front surface is curved and in the other the back, this will produce a shift in the nodal point of the filter of about its thickness and consequently a difference of I and therefore of size of image. The effect, however, is far too small to be detected.

I owe thanks to Mr. S. D. Chalmers and to Dr. F. E. Ross for assistance with some of the problems discussed in this paper.

Research Laboratory
EASTMAN KODAK COMPANY
Rochester, N. Y.
December, 1916.

A PHOTOCHEMICAL THEORY OF VISION AND PHOTOGRAPHIC ACTION[1]

By P. G. NUTTING

While the photochemical response to light has been quite extensively studied and a number of fundamental laws arrived at experimentally, no attempt at a comprehensive general theory of photo-chemical action appears to have been made. Such a general theory encounters at the outset the gravest mathematical difficulties in the nature of discontinuities and insoluble differential equations, and it may be found that no general theory is possible or not simple enough to be useful. In this paper the known general laws are summarized and interpreted and a general theory outlined.

Within the three great fields to which photochemical theory is applicable—namely vision, photography and the chlorophyll reaction in living plants—visual and photographic sensitometry have already supplied much of the required data, but very little is known of the laws of conversion of energy by plants. A notable contribution to the theory of the photographic reaction was made by Hurter and Driffield in 1890 (Jour. Soc. Chem. Ind., IX., 1890, p. 455), but their theory does not cover the failure of the Reciprocity law, the threshold effect, and the intermittency departure, nor is it applicable to heterogeneous media. In 1916, Houston published a resonance theory of vision and arrives at a visibility function corresponding fairly well with observed data.

A general theory of photochemical action is a relation between the effect produced and the intensity, quality and time of action of the light producing it. In the photographic reaction, the effect is measured by the mass of silver in the developable latent image. The specific action of the light waves or x-rays is to produce nuclei (electrons) which permit the precipitation of metallic silver after solution of the silver halide in weakly reducing solvents. In vision, a somewhat similar action is assumed, the ionization produced by the light affecting the nerve ends according to their number, but with a metabolic restoring action constantly in operation. In the case of simple chemical synthesis or reduction, the action of the light is probably a breaking or closing of chemical bonds.

[1] Presented before the Optical Society of America at the New York meeting, held December 28, 1916.

For the sake of concreteness, the course of the photographic and visual reactions is reproduced in the curves of Fig. 1 as experimentally determined.

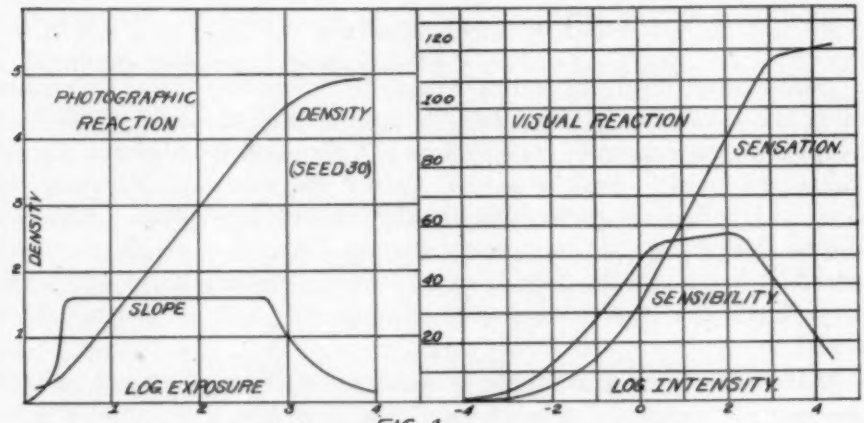


FIG 1

(Photographic and Visual Reaction)

The ordinates of the photographic curve are silver mass or optical density (which is proportional to mass), the lower curve being its derivative. In the curve for vision, the lower curve, photometric sensibility, is that determined experimentally, while the upper curve, representing sensation, is obtained by integration. The abscissae are log exposure in the first case and log intensity in the second. A close general similarity may be noted, namely a toe, a straight line portion and a shoulder. At very low intensities, the effect produced is proportional to the exposure, indicating a sensibly constant amount of reacting substance. In the central straight line portion, the effect is a linear function of the abscissa, hence each increment to the effect is proportional to the fractional increment to the abscissa. The upper horizontal portion of each curve, of course, represents equilibrium.

In the case of vision, there is a definite threshold at a flux density of about 2×10^{-5} lumen per mm^2 (on the retina) below which no effect is produced. This limitation is probably due to a failure to affect the nerve ends rather than to a cessation of chemical effect by the light. In the photographic and chlorophyll reactions, no absolute threshold has been observed below which no reaction has been produced by a sufficiently extended exposure, but in the photographic case the reciprocal relation

between intensity and time breaks down at an intensity of about 0.01 meter candle or 10^{-8} lumen per mm^2.

The Bunsen-Roscoe Law. Bunsen and Roscoe, working on mixed gases, discovered and formulated the law that the chemical effect produced by light is proportional to both the intensity of the light and to the time of exposure, a constant amount of reacting substance being assumed. This law holds without exception, as far as known.

The Reciprocity Law. This law states that the photo-chemical effect of increasing intensity alone is precisely the same as of increasing time of exposure alone or, in other words, that the effect is proportional to the product of intensity $\times$ time. This law holds very well except at very low ntensities, and in that region breaks sharply and completely. According to the so-called Schwarzschild law, the effect is proportional not to $I \times t$ but to It^p where p differs slightly from unity. Supposed verifications of the law have been subject to experimental errors of such magnitude, however, that it can hardly be regarded as proven. The reciprocity law and its failure are highly significant in any hypothesis as to the mechanism of the effect of light. The reciprocity law distinctly negatives any statistical theory of photo-chemical absorption whereby the number of molecules affected is related directly to the ease of their breakdown or combination. On the other hand, it is in line with the fundamental law of momentum and with the similar fundamental law of photo-electricity.

The Intermittency Law. According to this law, the integrated effect of a succession of short exposures is equivalent to a single exposure of the total time. In vision, the intermittency law is called Talbot's law. Under conditions such that the reciprocity law holds the intermittency law amounts to a statement that there is no elastic recovery between exposures. Both laws fail at very low exposures and high frequencies, but both hold for moderate and high intensities.

The Law of Molecular Opacity. Light can effect molecular changes only in those molecules which themselves absorb the light. Hence 1) no photo-chemical action can occur in a perfectly transparent medium, 2) no photo-chemical action can occur in a body which is merely turbid, scattering light without absorbing it, 3) no such action can occur in an absorbing medium in which the absorbing atoms or molecules do not form integral parts of the molecules to be affected by light. This law holds for each component of mixed media receiving mixed radiation, being a ***conditio sine qua non*** in all known cases.

Other significant characteristics of photo-chemical absorption may be noted. First, that the transformation of light into chemical energy is an extremely retarded one. But a small fraction of the total light energy is transformed in any case, and that which is is taken up very gradually; the major portion (usually over ninety per cent.) of the light is either passed on or merely changed into heat energy. It might well be expected that photo-chemically active bodies would merely sop up the light until saturated and then pass it completely, while, on the contrary, such bodies appear to be fundamentally incapable of taking up more than a small fraction and that not rapidly.

Molecular theory would account for this characteristic on the supposition that, while the light energy absorbed by each molecule causes an immediate accession of energy in a certain set of modes of motion, the resultant chemical change is brought about by quite another set of modes of motion, and that energy can be communicated from one set to another only very slowly. Such an hypothesis would be quite in accord with accepted molecular theory and would suffice to explain the very limited maximum rate of transformation of light into chemical energy. Any assumption of a direct change of light into chemical energy would involve immediate local saturation.

Without any assumption as to the nature or mechanism of absorption, homogeneous radiation penetrating a body in the direction z decreases in intensity (I) according to the law (Beer's law),

$$I = I_o e^{-kz}. \tag{1}$$

This is equivalent to the statement that the fractional decrease in intensity (dI/I) in any layer (dz) is a constant $(-k)$. This law does not apply in general to mixed radiation nor to bodies whose absorption (measured by k) varies with the depth. If two or more kinds of absorption co-exist in the same body, then Beer's law applies to each separately, $k = k_1 + k_2 +$ etc., being a purely additive quantity.

Owing to the enormous velocity of light, the time required for it to attain an equilibrium distribution is negligible in all ordinary problems. The rate at which energy is absorbed in any given layer is measured by $I_o - I$. If E represents energy and t time, then the energy dE absorbed in any layer extending from $z = o$ to $z = z$ in the time dt is $dE = (I_o - I)\,dt$ or $dE = I_o(1 - e^{-kz})\,dt$, assuming only conditions such that Beer's law holds.

In photo-chemical absorption, the concentration of the reacting substance and therefore the absorption coefficient (k) is constantly changing

and it is convenient to write the above energy equation in different form, due originally, I believe, to Hurter and Driffield. Let V be the total quantity per unit area of matter capable of being reacted upon by the light in question and let v be the quantity already acted upon. But, by Beer's law, the absorptive index k is proportional to the quantity of absorbing matter per unit area in a layer of given thickness. Hence, we may substitute hv for k, h being the constant of proportionality.

The energy absorbed in time dt by the unaffected absorbing matter is

$$(dE)_V = I_o\,(1-e^{-hVz})dt, \tag{2}$$

while that absorbed by the matter already affected is

$$(dE)_v = I_o\,(1-e^{-hvz})\,dt.$$

The difference between the quantities of absorbed energy is that used up in producing the chemical change of edv, where e is the amount of energy required to affect v by unity. Hence, by subtraction,

$$edv = I_o\,(e^{-hvz}-e^{-hVz})\,dt. \tag{3}$$

This is the fundamental equation of photo-chemical action in its simplest form. Additional terms are required to represent threshold effects and restoring actions, but, as written, the equation represents ordinary reactions very well indeed. Inactive absorption is expressly ignored on both sides of the equation (3).

As an equation in v and t_s (3) is readily solved for any constant thickness z. Write

$$e^{-hvz} \equiv Q, \qquad e^{-hVz} \equiv Q_0 \text{ and } \frac{I_o dt}{e} \equiv dE.$$

Q is transmission, while $I_o dt/e$ is the incident energy measured in terms of e as a unit. The substitution in (3) of the variables gives:

$$\frac{dQ}{Q\,(Q-Q_o)} = -hz\,dE \tag{4}$$

of which the solution is

$$\frac{I}{Q} = \frac{I}{Q_0} - \left(\frac{I}{Q_0} - I\right) e^{-hzEZ_o}.$$

Hence,

$$v = \frac{I}{hz} \log \left[\frac{I}{Q_0} - \left(\frac{I}{Q_0} - I\right) e^{-hzEQ_o}\right]. \tag{5}$$

Equation (5) is a complete solution of (3) and differs from Hurter and Driffield's equation only in being slightly more general. To reduce it to a form more directly applicable to the photographic problem, we note that mass per unit area v is number of developable silver grains per unit area times average mass per grain. But, it has been shown by statistical methods[1] that photographic density (=−log transmission) is equal to area of grain times number of grains per unit area, hence density

$$D = A \log (B - (B-1)\, e^{-hzE/B}, \tag{6}$$

where $A = \frac{\text{area of grain}}{\text{mass per grain}} \times hz$ and $B = \frac{1}{Q}$ is the opacity due to the exposable silver halide grains alone. For $E = 0$, $D = 0$, for $E = \infty$, maximum exposure, $D = A \log B = aN$. If density be plotted against log exposure as in Fig. 1, the curve has a maximum slope (γ) when $\frac{dD}{d\,(\log E)}$ is a maximum. This occurs at an intermediate value of E given by the equation (C being written for hz/B)

$$(EC-1)\ (1-(B-1)e^{-cE}) = B-1$$

This cannot be algebraically solved for E in general. For thin layers and low opacity B, $E_{max} = 1/hz$ and the maximum slope approximates proportionality to lamh, area of grain, mass of grain, and absorption coefficient. In ideal development, each grain of developable halide (latent image) is converted into a grain of metallic silver and the result obtained applies, with proper interpretation, to the developed image.

The equations for simple photo-chemical action developed above apply only to the case of homogeneous radiation absorbed by homogeneous bodies in which there is no elastic, retarding or restoring action. When such actions exist, these equations must be modified and extended.

At the threshold of any photo-chemical reaction, where the light intensity is barely sufficient to cause a reaction, the cause of the failure of the simple equations appears to be, in some cases, an elastic or viscous reluctance to the breaking of bonds within the molecule. In such cases the limiting intensity is exceedingly low, due perhaps to both the weakness of the bond and to the cumulative action of the light energy. In photography, the reciprocity law breaks abruptly at an intensity of about 0.01 meter candle for white light. The failure of the intermittency law is attributable to the same causes. Vision near the threshold is of such a nature as to indicate both elastic and retarding action. The restoring action connected with

[1] *Phil. Mag.*, Sept. 1913, p. 423.

reformation of visual substance is probably inoperative at the threshold because of saturation conditions. Near the threshold of vision the effect is proportional to total energy rather than intensity, hence we conclude that the simpler photo-chemical laws hold.

When a threshold exists, the simple linear relation between dE/dt and I_o given by (2) no longer holds in the region just above the threshold. Graphically, this departure may be represented as in Fig. 2.

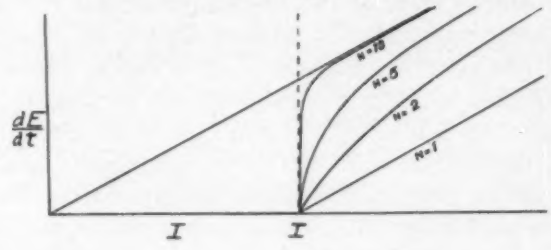


FIG. 2.

The value of dE/dt rises more or less abruptly at the threshold intensity I_t but for higher intensities approaches the linear relation. To represent this, I_o in equations (2) must be replaced by some function of that variable. Synthetic function theory indicates that the desired function is a power difference such that (2) is replaced by

$$dE = (I_o^n - I_t^n)^{\frac{1}{n}} (1 - e^{-h\nu s})\, dt. \tag{7}$$

Making the corresponding modification in the concentration equation (3) leads to an integral equation similar to (5) in form, but in which the E of the final exponential term is

$$E \equiv (I_o^n - I_t^n)^{\frac{1}{n}} \frac{t}{e}.$$

The effect of this modification on the $D - \log E$ curve is to condense it in the region near and just above the threshold.

Intensity and time no longer enter the equation as a simple product, hence there is a departure from the reciprocity law near the threshold. The effect of the light is the same function of time as before, but the intensity effect falls off rapidly near the threshold, the rate of falling off depending upon the magnitude of the parameter n.

In the theory of the response of the photographic emulsion to light, given previously, equality is assumed between the energy used in producing a change of concentration and the difference between the energy absorption

of all the active substances and that of the affected substance. In the case of vision, no such equation holds, for, during exposure to light, the retina itself is constantly restoring the affected phase to the unaffected condition. During a continued, steady exposure there is no net change in the concentration of any phase. The effect of the exposure is just counteracted by the restoring action of the retina, the position of the balance between the two determining both the amount of the reaction and the sensibility to reaction.

In the unbalanced state of the retina, during the initial stages of exposure or rest, both concentration and sensibility are changing. Using a notation similar to that previously used, let V be the visual substance per unit area approached after prolonged rest, and v that of the substance affected by light. As before, edv is the energy required to change the concentration v by the amount dv. The metabolic action of the retina is proportional to the difference between the concentrations V and v and to the time dt or $m(V-v)$. The remainder of the equation,

$$edv - m(V - v)\,dt = I_o(e^{-hvz} - e^{-hVz})\,dt, \tag{8}$$

is as before the difference between the energies absorbed by all the visual substance and the affected portion of it, per unit area and time dt. Equation (8) is unfortunately not integrable without simplifying assumptions.

In any *steady state*, the change dv is zero and (8) reduces to

$$mv - I_o e^{-hvz} = m V - I_o e^{-hVz} = \text{constant} \equiv C, \tag{9}$$

which is algebraically insoluble in general. For moderate and high intensities I_o, the concentration v is small and

$$v = \frac{C}{m + I_o hz} \tag{10}$$

that is, the sensibility varies inversely as the intensity in a rather complex manner when the adaption is complete.

Another case of considerable practical interest is that in which v is small while the limiting concentration V is relatively large. This is true in the range of ordinary working brightnesses. In this case, the second member of (8) reduces to $I_o kzvdt$, and (8) may be written

$$e\frac{dv}{dt} = mV - (m + I_o kz)^{v}. \tag{11}$$

$$= a - bv, \text{ say.}$$

This integrates into

$$\log\frac{a-bv}{a-bv_o} = bt, \text{ or} \tag{12}$$

$$v - v_o = (1 - e^{-bt}) \times \text{constant},$$

v_o being the value of v at the time t = o. An equation of this form has already been arrived at empirically to fit the data on rate of adaptation. In adaptation from a lower to a higher intensity, the constant b in (12) has the value of the parenthetical term in (11), in adaptation from a higher to a lower intensity, b = m, since I_o = o. Since b is larger in the first case, adaptation is more rapid. Experimental evidence supports this conclusion. The variation of sensibility with wave length ("visibility") is taken account of in the absorptive index k of which the parameter b is a function.

Of chief practical interest in vision is the relation between the brightness sensation and the luminous flux producing it in the steady state; that is between v and I_o in the integral of (8) when time $t = \infty$. While equation (8) cannot be integrated directly, a relation consistent with it may be obtained from the known data on photometric sensibility.

Sensibility to differences in brightness is dv/d (log I). This is proportional to the reciprocal of the Fechner fraction for which an expression has been found,* namely:

$$\frac{\delta I_o}{I_o} = P = P_m + (1 - P_m)\, I_o^n\, I_t^{-n}$$

Hence

$$\frac{dv}{d(\log I_o)} = \frac{1}{P_m + (1 - P_m)\, I_o^{-n}\, I_t^n} \tag{13}$$

from which

$$v = \frac{1}{n\, P_m} \log\left[1 + P_m\left(I_o^n\, I_t^{-n} - 1\right)\right] \tag{14}$$

and therefore

$$e^{-P_m v}\left(1 + P_m\left(I_o^n\, I_t^{-n} - 1\right)\right)^{\frac{1}{n}} = \text{constant}$$

which is consistent with (8) for dv/dt = o and P_m = hz.

Equation (14) holds from the threshold of vision up to the highest intensities for which Fechner's law holds. The quantities n and I_t are functions of both time and wave length, P_n is independent of both. The relation obtained is slightly more general than the corresponding one contained in equations (5) and (7) applicable to photographic action at very low exposures.

* *Physical Review*, Feb. 1907, p. 208.

WESTINGHOUSE RESEARCH LABORATORY
East Pittsburgh, Pa.
January, 1917

SYMBOLS AND DESIGNATIONS FOR GEOMETRICAL OPTICS

By HERMANN KELLNER

I

The arbitrary and often inconsistent way of designating different magnitudes which is practiced by different writers on geometrical optics is confusing to the beginner and annoying to the advanced reader. Every author uses his own system of designation and there is hardly a single one who is not inconsistent.

In accepting the editorship of this journal the possibility occurred to the writer of bringing about, at least in the articles published in this journal, a uniform system of designation and, with this in mind, he approached Dr. Southall for his views on the subject. Dr. Southall has given this matter a good deal of thought, and expressed himself to be in perfect accord with the idea, and as a result of several conferences we offer the following suggestions.

It seemed, in the first place, desirable that the type of the letter give some indication of the nature of the magnitude it represents. We therefore have reserved Roman capital letters for points, lower case italics for magnitudes, italic capitals for reciprocals and Greek lower case letters for angles. Two exceptions from these rules could not be avoided, viz. the use of the λ for the wave length and of the Δ for the optical interval, these departures being justified by universal usage.

The same letter or designation is used for conjugate or similar points or magnitudes on both sides of a refracting surface. They are distinguished from each other by a ($'$) which denotes that the magnitude so marked lies on the side of the optical system away from the side from which the light comes. For instance: f and f', n and n' are the focal lengths and refractive indices respectively on the left and the right side of the refracting surface, and α and α' the slope angles before and after refraction. Elements of compound system are distinguished by subscripts counting from the left to the right through the system, for instance r_1, r_2, r_3, . . . r_k would be the first, second, third and k^{th} radius of the system. The symbols referring to the whole system in contradistinction to its components are used without subscripts.

The light is always assumed to enter the optical system from the left side, and the direction from left to right is assumed to be positive.

Distances will be counted positive in the direction of the light, negative in the opposite direction.

Radii are measured from the vertex of the surface to the center. This makes radii of surfaces, convex towards the incident light, positive, while the radii of surfaces concave toward the incident light will be negative.

Object and image distances are measured from the vertex of the surface,

Focal lengths from the principal points.

Ordinates in object and image planes are positive above and negative below the axis.

Slope angles above the axis are positive when they converge in the direction of the light, negative when they diverge. They take the opposite sign when below the axis.

Designations of Points (*Roman capital letters*)

Focal points	F and F′
Principal points	H and H′
Nodal points	N and N′
Conjugate object and image points	O and O′
Centre of spherical refracting surface	C
Vertex of spherical surface	V

Symbols of Magnitudes (*Italic lower case*)

Index of refraction	n, n'
Wavelength	λ
Radius of curvature of refracting system	r
Focal lengths of optical system	f and f'
Distance from vertex V to focal point F	v and v'
Distance between vertices of two refracting surfaces	d
Distance between vertex of refracting surface and object or image point	u and u'
Incidence height	h
Abscissae measured from principal points	b and b'
Reduced distance between two principal planes	c
Abscissae measured from focal points	x and x'
Ordinates of conjugate points	y and y'

Symbols of Magnitudes (Italic lower case) Continued

In a compound system the distance between the posterior focal point of the first component and the anterior focal point of the second component, the optical interval,	Δ
Distance between the focal point of a compound system and the posterior focal point of the second component	e

Reciprocals of magnitudes
are to be designated in the corresponding capital italics thus:

$$U = \frac{1}{u} \qquad X' = \frac{1}{x'} \qquad F = \frac{1}{f} \qquad V = \frac{1}{v}$$

or better still, following Gullstrand, let

$$U = \frac{n}{u} \qquad X' = \frac{n}{x'} \qquad F = \frac{n}{f} \qquad V = \frac{n}{v}$$

denoting by capital italic letters the so-called reduced lengths.

F and V represent quantities of great importance in ophthalmic optics, F is the so-called refraction of a lens, V its vertex refraction. Both are expressed in diopters (dptr) when f and v are measured in meters.

Angular magnitudes are denoted by lower case Greek letters.

Slope angle	α and α'
Angle of incidence and refraction (or reflection)	ϑ and ϑ'
Central angle	φ

Magnifications

Lateral magnification	Y
Axial or Depth magnification	X
Angular magnification	A

The following diagrams are given to supplement the foregoing list of designations.

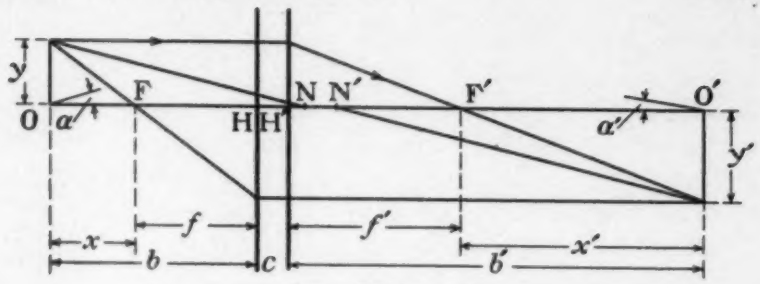


Fig. 1
An object y, imaged by an optical system represented by the principal points H and H', the nodel points N and N', and the focal points F and F'.

$$x \cdot x' = f \cdot f'$$

$$\frac{\mathrm{d}\,x'}{\mathrm{d}\,x} = -\frac{f \cdot f'}{x^2} = -\frac{x'}{x} = \mathbf{X}$$

$$\frac{y'}{y} = \frac{f}{x} = \frac{x'}{f'} = \mathbf{Y}$$

$$\frac{\tan \alpha'}{\tan \alpha} = -\frac{x}{f'} = -\frac{f}{x'} = \mathbf{A}$$

$$y'\, n' \tan \alpha' = y\, n \tan \alpha$$

$$\mathbf{Y\,A} = \frac{n}{n'} = \frac{f}{f'}$$

$$\frac{\mathbf{X\,A}}{\mathbf{Y}} = 1$$

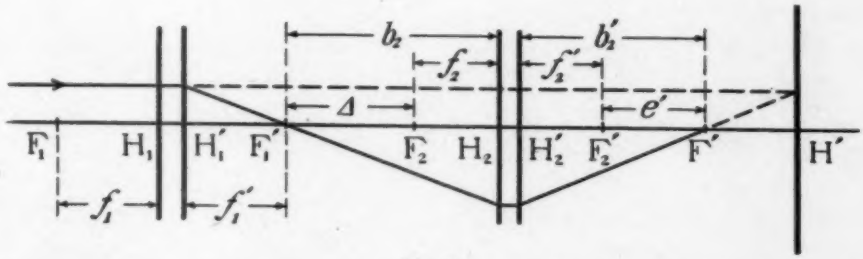


Fig. 2
A compound system composed of two elements represented by the principal points H_1, H_1' and H_2, H_2', and by the focal points F_1, F_1' and F_2, F_2', the distance F_1', F_2 being the optical interval Δ; object point at $-\infty$.

$$f' = \frac{f_1' f_2}{\Delta} \qquad f = -\frac{f_1 f_2'}{\Delta}$$

$$e' = -\frac{f_2 \cdot f_2'}{\Delta} \qquad e = \frac{f_1 f_1'}{\Delta}$$

$f' = b'$ because $u = \infty$

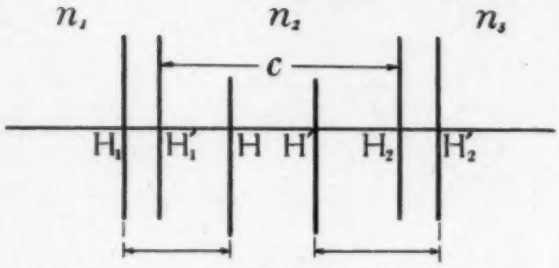


Fig. 3
The principal planes H, H′ of a compound system, and those of its components H_1, H_1' and H_2, H_2' in their relative locations.

The refracting power of a compound system (Fig. 3) composed of the components having respectively the powers F_1 and F_2 and having a separation between their principal points H_1' and H_2 equal to c, will be

$$F = F_1 F_2 - F_1 F_2 c$$

where $c = \frac{H_1 H_2}{n_2}$.

The location of the principal points of the whole system with respect to H_1 and H_2' follows from

$$\frac{H_1 H}{n_1} = c \frac{F_2}{F} \qquad \frac{H_2' H'}{n_3} = - c \frac{F_1}{F}$$

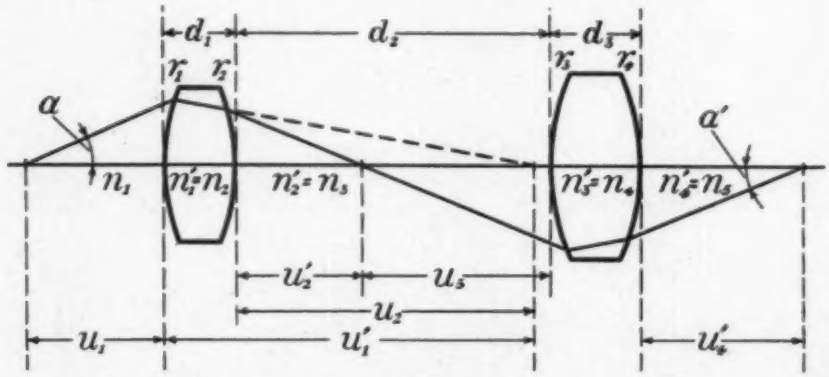


Fig. 4
Optical imagery through a compound system consisting of two positive lenses of thicknesses d_1 and d_3 respectively, separated by a distance d_2.

In Fig. 4 is

$$u_2 = u_1' - d_1$$
$$u_3 = u_2' - d_2 \text{ etc.}$$
$$n_1' = n_2$$
$$n_2' = n_3 \text{ etc.}$$

when $u_1 = \infty$, u'_4 becomes v'.

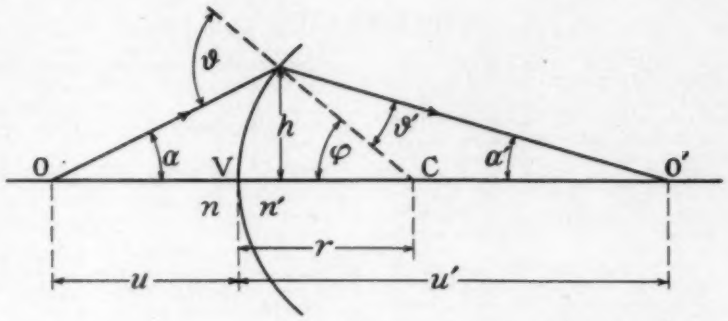


Fig. 5
Designation of the magnitudes essential in trigonometric calculation of the path of a ray refracted by a spherical surface, the ray path lying in the plane of a principal section.

Fig. 5 is self-explanatory and requires no comment.

(To be continued)

Optical Society of America, Annual Meeting, New York City, December 28, 1916

The first Annual Meeting of the Optical Society was held in New York City, December 28, in Room 301 of the Physics Building, Fayerweather Hall, under the auspices of the A. A. A. S., and the following papers were read:

L. T. Troland, Harvard University.
Notes on the Theory of the Visual Receptor Process.

M. B. Hodgson, Eastman Kodak Co.
Compensating Filters for Use in Spectroscopy.

Edw. P. Hyde and W. E. Forsythe, Nela Research Laboratory.
The Quality of Light Emitted by Various Sources as Indicated by Their Color Temperatures.

F. E. Ross, Eastman Kodak Co.
1) The Photography of Star Images.
*2) The Effects of Exposure, Developer and Development on Photographic Resolving Power.

E. L. Nichols, Cornell University.
Optical Research.

F. E. Wright, Geophysical Laboratory.
1) The Petrographic Microscope in Applied Optics.
2) The Determination of Anisotropism in Opaque Substances.

H. C. Lord, Emerson McMillan Observatory.
On Designing a Symmetrical Photographic Doublet Consisting of Four Separated Lenses.

Herbert E. Ives, United Gas Improvement Co.
The Use of Lippmann Films as Sources of Monochromatic Light in Photometry and Optical Pyrometry.

H. Kellner, Bausch & Lomb Optical Co.
The Determination of the Chromatic Aberration of the Eye for Different Zones of the Pupil.

L. A. Jones, Eastman Kodak Co.
The Fundamental Hue Scale and the Retinal Sensibility to Hue Differences.

*C. W. Frederick, Eastman Kodak Co.
Variants of the Seidel Aberrations in the Four Piece Separated Type of Objective.

Julian Blanchard, Eastman Kodak Co.
The Brightness Sensibility of the Retina.

P. G. Nutting, Westinghouse Research Laboratory.
1) A Photochemical Theory of Vision and Photographic Action.
2) A Proposed New Method of Classification for Optical Glass.

* Read by title.

ABSTRACTS

A DIRECT READING PRECISION REFRACTOMETER WITH UNIFORMLY DIVIDED SCALE. G.W. Moffitt, Phys. Rev., Dec. 1916. Dr. G. W. Moffitt describes in the *Physical Review* for December a new refractometer for liquids. It employs the well known method of measuring the effect on the focal length of a lens of partially or wholly immersing it in the liquid. The principal variation from former types lies in providing an uniformly divided scale reading graduated directly in indices of refraction.

At C is a target; the liquid of unknown index of refraction, n, is introduced at T between a plane parallel glass plate and a lens L. L is a positive lens of focal length f, I is the final image of C, and p its distance from L. Then if T coincides with the lower principal focus of L,

$$n = \frac{R}{p}P + \left(1 - \frac{R}{u} - \frac{R}{f}\right)$$

where R is the distance CT, and u is the distance from T to the image of C formed by the lens L. This equation is linear in n and p for any constant value u.

The magnification is constant and equal to $\frac{f}{u}$. This makes possible a method of setting which consists in adjusting p until coincidence is obtained of the size of image and I with a predetermined value on an eyepiece scale.

Spherical aberration, curvature of field and distortion are discussed and dismissed as unlikely to affect the results, possibly with a little too much haste so far as spherical aberration is concerned. Chromatic aberration will cause no trouble if monochromatic light be used. The employment of the instrument for the determination of the refractive indices for different colors involves the employment of achromatic lenses.

The order of accuracy attainable should reach the third place of decimals with a probable error of one or two units.

BLACK BODY BRIGHTNESS EQUATIONS BASED ON EQUIVALENT WAVE LENGTHS. Herbert E. Ives and E. F. Kingsbury, Phys. Rev. v. 8, 323–325, Sept. 1916. The product of the spectral energy emitted by a perfect radiator and the visibility of radiation, when integrated, gives a simple relation between the luminosity and the temperature of such a radiator. The Rasch equation for this luminosity assumes an equivalent wave length for the luminous spectrum which is independent of temperature and leads to the luminosity equation

$$\log L = a + \frac{b}{T},$$

in which L is luminosity, T temperature, a and b constants.

Making no simplifying assumptions, the complete theory, as developed by Nutting, gives

$$\log L = a + b \log \left(\frac{c}{T} + 1\right).$$

The author's show that the simpler formula

$$\log \left(\frac{L}{24000}\right) = -\frac{1}{0.00003795T + 0.0025}$$

represents the brightness of a perfect radiator within the range from 1200° to 3000° to about half a per cent. The brightness is in light watts per steradian per unit solid angle.

LUMINOUS EFFICIENCY OF THE SOLAR RADIATION. H. E. Ives, Trans. I. E. S. XI, 888–890, Nov. 1916. For several years the U. S. Weather Bureau has been obtaining data on both solar illumination and solar radiation at four different stations at all seasons of the year and various zenith distances. The work is in charge of Dr. H. H. Kimball and appears regularly in the Monthly Weather Review. In the Trans. I. E. S., May 1916, Dr. Kimball publishes a summary of his data, the solar illumination being in foot-candles, the radiation in gram calories per min. per cm². Ives reduces this to lumens and watts by the relations: 1 foot-candle = 0.001076 lumens per cm² and 1 gram. cal. per minute = 0.0697 watt. Hence, Kimball's ratio of illumination to radiation, multiplied by 0.01545, will be reduced to lumens per watt. These mean data are:

	Lumens per watt.
Solar zenith distance 48.3°	86.5
" " " 66.5	78.9
" " " 73.5	71.1
Diffuse sky radiation	137.3
Over cast sky	96.5

A MODIFICATION OF THE HILGER SECTOR PHOTOMETER METHOD. H. E. Howe, Phys. Rev. VIII, 674–688, Dec. 1916. In the Hilger method, as used in the ultra violet sector spectrophotometer supplied by that firm, the divided beam from a single source passes through two rotating sector disks on its way to the slit of a quartz spectograph. One sector is of variable angle, and before the other sector is placed the body whose absorption is to be determined. The author finds, for four different plate emulsions, that the intensities, as determined by equal blackenings of the plate, an inversity proportional to the sector openings, hence, the correction supposed to be applicable to each brand of plate used is quite unnecessary.

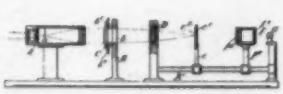
For obtaining a spectrum fairly continuous in the ultra violet, essential in determining spectral absorption, the author improves the method of V. Henri (Ph. Zeit., 14, p. 516, 1913) of using an aluminum spark under water. A very heavy discharge through a Tesla coil is the essential feature. This coil is operated by an oil transformer directly on the AC mains. A capacity of about

0.02 m.f. is used. Distilled water was found necessary. The spectrum obtained is excellent, being considerably superior to the condensed cadmium spark.

THE ELECTRIC ARC IN VAPORS AND GASES AT REDUCED PRESSURES. W. A. Darrah, Trans. I. E. S., XI, 933–5, Nov. 1916. (Abstract of a paper appearing in full in Trans. Am. Electrochemical Soc.) Ordinary arc lamps operate in atmospheres rich in oxygen and give accumulations of non-volatile oxides and carbon. Longer life is to be secured by avoiding such products. Tungsten electrodes waste away very little compared with carbon. The best vapors are such that a) the metallic part gives a very luminous white flame, b) the acid part gives a stable arc at low voltages and c) assists recombination in the flame zone, thus minimizing deposits.

Various tetrachlorides were found very suitable when tightly enclosed in small glass bulb surrounding the electrodes. The outlook for a closed cycle high efficiency arc is considered promising.

PATENTS



LENS CENTERING AND MEASURING INSTRUMENT in which A is an observing telescope with cross hairs at a, C the lens to be centered, D a standard lens in fixed position, e a support carrying a single wire or line across a transparent opening, and F a support carrying another lens f_2 and another single line target f which is so disposed as to be imaged by the lens f_2 at some point in front of the latter and form with the wire at C a pair of cross-hairs. The standards C and F are adjustable with respect to each other and with respect to D, so that it will always be possible to find positions for C and F such that the images of the two target lines will coincide with the focal points of the two corresponding meridians of the lens C and an observer looking through the telescope will see a sharply defined cross whether the lens C be astigmatic or not.

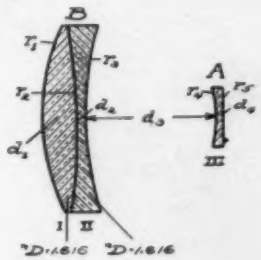
If C is a simple lens having the same power in all meridians, then the image of the target line f formed by the lens f_2 must lie in the same plane as the target line carried on the support e. If C is astigmatic, the image of f will lie in front of or back of the plane of e so that even for astigmatic lenses it is possible to get a sharply defined image of the cross.

By shifting C the cross may be brought into coincidence with the cross-hairs of the telescope and the lens is then centered with respect to the optical axis of the instrument.

A scale E makes it possible to use the instrument for measuring the powers of the two meridians of the lens.

Six Claims.

Gustav A. H. Kellner, Rochester, N. Y.
U. S. P. No. 1,204,030.
Application filed July 14, 1916.
Patent issued Nov. 7, 1916.



A LENS SYSTEM FOR GALILEAN TELESCOPES which provides a larger field of view free from objectionable astigmatism, distortion, and curvature of field than has been attainable with prior constructions. Chromatic correction is attained by combining in the objective lens a crown glass and a flint glass of very nearly the same index of refraction for one color.

Construction data are given as follows:

Kinds of glass:

n_{DI}	= 1.6119	ν_I	= 58.8
n_{DII}	= 1.6113	ν_{II}	= 37.1
n_{DIII}	= 1.6181	ν_{III}	= 36.5

Radii thicknesses and distances:

r_1	= 35.0 mm
d_1	= 1.5 mm
r_2	= 150.0 mm
d_2	= 1.5 mm
r_3	= 86.9 mm
d_3	= 32.7 mm
r_4	= 27.8 mm
d_4	= 1.0 mm
r_5	= ∞

There are two claims.

Gustav A. H. Kellner, Rochester, N. Y.
U. S. P. No. 1,197,742.
Application filed Feb. 26, 1912.
Patent issued Sept. 12, 1916.

PHOTOGRAPHIC OBJECTIVE

A triplet lens consisting of two positive lenses which enclose a negative lens is combined with a negative meniscus lens in such a way that this negative lens corrects the aberration of the triplet without reducing the relative aperture to a considerable extent.

Three claims.
C. H. Florian, Paris, France.
U. S. P. No. 1,168,873.
Application filed June 4, 1913.
Patent issued Jan. 18, 1916.

PHOTOGRAPHIC OBJECTIVE

A combination of four single lenses, two of which are positive lenses, symmetrically placed in the combination, but unsymmetrical in form, enclosing two biconvex lenses symmetrical in form of equal curvatures the eight lens surfaces being ground to three common radii.

Two claims.
Gilbert T. Day and Max Zwillinger, Rochester, N. Y.
Patent No. 1,110,128.
Application filed July 31, 1915.
Patent issued Nov. 16, 1915.

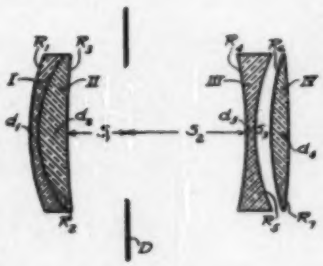
LENS FOR PLACING BEFORE A PHOTOGRAPHIC OBJECTIVE

The invention rests on the knowledge that the part to be played by the lens with regard to the objective in front of which it is placed, is that of a spectacle lens placed before a short-sighted eye. It is essential that the astigmatic aberrations in the lens are reduced to a minimum, as not to interfere with the corrections of the objective. Two examples are given which increase the focal length of an objective of 100mm to 137mm and 165mm. Both are meniscus lenses facing the objective with their concave surface, the radius of the said concave surface is at least one and a half times and not more than four and a half times as great as the distance of said surface from the entrance pupil of the objective.

One claim.
Guido Mengel, Dresden, Germany, Assignor to Carl Zeiss, Jena.
U. S. P. No. 1,119,731.
Application filed, Aug. 4, 1915.
Patent issued Jan. 25, 1915.

TELESCOPE in which means are provided to decenter the exit pupil with respect to the optical axis. Application is made to the case of the single field, two observer telescope described in U. S. P. No. 991,652 in which one-half of the cone of light is deflected by a reflector into an eyepiece making an angle with the principal axis of the instrument while the other half of the cone of light proceeds along the principal axis and through the principal eyepiece. Back of each eyepiece there is formed then a semi-circular exit pupil whose straight boundary line intersects the optical axis. The patent covers means for centering this partial exit pupil on the optical axis specifying such mean as a pair of rotating prisms, a prism which may be inserted in the path of the light and withdrawn at will, and a lens so arranged as to be decentered at the will of the observer, any of which preferably must operate near the focal plane of the eyepiece.

Seven claims.
W. B. Rayton and G. A. H. Kellner, Rochester, N. Y.
U. S. P. No. 1,196,811.
Application filed Nov. 8, 1913.
Patent issued Sept. 5, 1916.



OBJECTIVE LENS

An objective lens system having an image distance of short length relatively to the equivalent focal length thereof, composed of a compound positive meniscus (I-II), a dispersive intermediate lens, (III), and a collective biconves lens (IV). The compound meniscus (I-II) is formed by cementing a negative meniscus of high refractive index to a positive meniscus of lower refractive index. The air space between (I–II) and (III) is considerably larger than that between (III) and (IV). Collective lens (IV) has a higher refractive index than dispersive lens (III), the latter must not be lower than 1.60. The focal lengths of the three members are for (I–II) not greater than five tenths of the equivalent focal length of the combined lens system, for (III) two tenths and for (IV) three tenths.

Four claims.
William Bielicke, Rochester, N. Y.
U. S. P. 1,202,021.
Application filed April 24, 1916.
Patent issued Oct. 24, 1916.